Math 533  
Unit 6 Overview

A. Activity

This week we will be investigating the delta-epsilon definition and conceptual foundations of the limit. Remember that all questions/tasks posed in red need to be completed and submitted. **Send your solutions directly to Rachael Welder via WebCT mail by Thursday, November 13, 2003.**

When Newton and Leibniz originally invented calculus towards the end of the seventeenth century, they did not start out by using epsilon-delta proofs. However, as rigorous calculus developed throughout the eighteenth century, the epsilon-delta idea became the preferred method for defining many complex calculus ideas including limits and sums of series.

Read the following article on the historical development of rigorous calculus. The article has been posted in the Unit 6 folder as a PDF file. Try not to get overwhelmed with the advanced mathematics embedded in this article. The author calls upon fairly high-leveled examples, but even if you don’t follow the details of each, you can still benefit from the main concepts of the article. It tells an excellent story of how and why rigorous calculus developed, leaving us with the epsilon-delta idea.

“Who Gave You the Epsilon? Cauchy and the Origins of Rigorous Calculus” by Judith V. Grabiner, Visiting Professor of History at the University of California, Los Angeles.

After reading the article, considering the following epsilon-delta definitions of limits:

<table>
<thead>
<tr>
<th>Definition</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \lim_{x \to a} f(x) = L ]</td>
<td></td>
</tr>
<tr>
<td>( 0 &lt;</td>
<td>x - a</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Definition</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \lim_{x \to +\infty} f(x) = L ]</td>
<td></td>
</tr>
<tr>
<td>( 0 &lt; f(x) - L &lt; \varepsilon. )</td>
<td></td>
</tr>
</tbody>
</table>
If you need some review regarding the epsilon-delta definition of a limit, there are several great applets, graphical tutorials, and related pictures on the web. You may find it useful to visit one or more of the following websites:

- [http://www.math.hmc.edu/calculus/tutorials/limits/figures.html](http://www.math.hmc.edu/calculus/tutorials/limits/figures.html)
- [http://www.math.montana.edu/frankw/ccp/calculus/estlimit/graphic/learn.htm](http://www.math.montana.edu/frankw/ccp/calculus/estlimit/graphic/learn.htm)

Choose one of the following limits and use the appropriate limit definition to prove (or thoroughly explain why) the limit of the function is zero:

a. \( \lim_{x \to 0} 2x = 0 \)

b. \( \lim_{x \to +\infty} \frac{1}{x} = 0 \)

Your “proof” should be more than an explanation that as \( x \) approaches 0 or infinity, \( f(x) \) also approaches or becomes arbitrarily close to 0. Rather, explain why the limits are 0 according to the appropriate limit definition. In the event that you need a little assistance with limits, several great examples and limit tutorials can be found on the web:

- [http://archives.math.utk.edu/visual.calculus/1/definition.6/index.html](http://archives.math.utk.edu/visual.calculus/1/definition.6/index.html)
- [http://archives.math.utk.edu/visual.calculus/1/definition.7/index.html](http://archives.math.utk.edu/visual.calculus/1/definition.7/index.html)

Now that we’ve reviewed the delta epsilon definition of limit, let’s turn our attention to one of the more complex features of calculus - facilitating the development of students’ understanding of limit. After teaching the following sum,

\[
\sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n = 2
\]

a student approaches you and asks you how this sum could possibly be a finite amount, when you are continually adding more and more positive numbers. She believes that this sum should keep increasing, since you are adding an infinite number of positive numbers. **How would you respond to this student? Why doesn’t this sum keep increasing?**

**How would you convince her that this sum really is equal to 2?** You might pose this question to others in your discussion group.
B. Discussion

Page 385 of our course text states:

*The beginning calculus student of today, looking at the representations of Oresme, probably would first think of the steepness of the velocity-time curve as a measure of the acceleration, the rate of change of velocity with respect to time. Only later is he likely to think of the area under the graph as a measure of distance; today the concept of the derivative is usually presented first in calculus courses, with the notion of the integral coming later. Those textbooks that reverse the roles and place the integral before the derivative in a sense have history on their side, inasmuch as integration preceded differentiation by about two thousand years.*

The focus of our Unit 6 discussion is on integration/differentiation.

*Why do we teach differentiation prior to integration, when historically the two ideas were developed in the reverse order? Are there benefits to teaching these concepts in this order? Would students benefit more from them instead being taught in their historical order?*

One of the goals of the MS program is that all MS students graduate with a broad view of K-12 issues in mathematics education. Even if you do not teach calculus, I encourage you to actively participate in this discussion. To assist you with this discussion, I’ve provided a few resources regarding the historical development of derivatives and integrals.

1. “*The Changing Concept of Change: The Derivative from Fermat to Weierstrass*” by Judith V. Grabiner, Visiting Professor of History at the University of California, Los Angeles.
   (This article is posted in the Unit 6 folder as a PDF file.)

2. [http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/The_rise_of_calculus.html](http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/The_rise_of_calculus.html)


5. “*The History of Calculus*” by Carl Boyer (pp. 376-403 of the course text).

Post at least one message addressing these questions (your message must make a direct reference to at least one of these resources) in the appropriate LIM Discussion folder by Saturday, November 15, 2003. Respond to the posts of at least two colleagues by Thursday, November 20, 2003.
As before, you will be placed into discussion groups. Please post some initial thoughts/questions about the problems in the appropriate folder (LIM1, LIM2, or LIM3). Also, feel free to share your thoughts about either of the activities. Keep in mind that the discussion groups are here to help you and for you to help others.

<table>
<thead>
<tr>
<th>LIM1</th>
<th>LIM2</th>
<th>LIM3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bundy</td>
<td>Bertelsen</td>
<td>Brohaugh</td>
</tr>
<tr>
<td>Smith</td>
<td>Macdonald</td>
<td>Stewart</td>
</tr>
<tr>
<td>Sinibaldi</td>
<td>Hodik</td>
<td>Beavin</td>
</tr>
<tr>
<td>McDaniels</td>
<td>Bertrand</td>
<td>Savage</td>
</tr>
<tr>
<td>DeWitt</td>
<td>Shultz</td>
<td>Bentley</td>
</tr>
<tr>
<td>O’Kelley</td>
<td>McCluskey</td>
<td></td>
</tr>
</tbody>
</table>

C. Lesson Plan

Chapters VII of your course text lists a variety of topics related to Calculus. Select one of these topics (or a topic of interest to you that’s related to Calculus) and an existing lesson plan from your classroom that utilizes the topic. Re-write your lesson plan so that the history of the topic is incorporated into the lesson plan. You will probably need to conduct some library or Internet research on your topic to gain an understanding yourself.

Your lesson plan should not be a lesson on history. Rather, it should be a lesson plan on a mathematical topic in which history is embedded. Your lesson plan should be brief (1-2 pages) and clearly outline your objectives, the sequence of activities by which students’ understanding of the topic is developed, and the historical content embedded in the lesson. Post a copy of your lesson plan as a discussion message (or an attachment to a discussion message) in the Calculus Lessons discussion folder by Thursday, November 20, 2003. You are not required to read others’ lessons, but I want to make the lessons available to the class. Read and enjoy the work of your colleagues.